CHAPTER 6

MOMENTUM

Newton's laws are useful for solving a wide range of problems in dynamics. However, there is one class of problems in which, even though Newton's laws still apply as we have defined them, we may have insufficient knowledge of the forces to permit us to analyze the motion. These problems involve collisions *between one object and another.*

In this chapter we will learn how to analyze collisions between two objects. In doing so, we will find that we need a new dynamic variable, called linear momentum. *We will see that the law of conservation of linear momentum, one of the fundamental conservation laws of physics, can be used to study the collisions of objects from the scale of subatomic particles to the scale of galaxies.*

6-1 COLLISIONS

In a collision, two objects exert forces on each other for an identifiable time interval, so that we can separate the motion into three parts: before, during, and after the collision. Before and after the collision, we assume that the objects are far enough apart that they do not exert any forces on each other. During the collision, the objects exert forces on each other; these forces are equal in magnitude and opposite in direction, according to Newton's third law. We assume that these forces are much larger than any forces exerted on the two objects by other objects in their environment. The motion of the objects (or at least one of them) changes rather abruptly during the collision, so that we can make a relatively clear separation of the situation before the collision from the situation after the collision.

When a bat strikes a baseball, for example, the time between the beginning and the end of the collision can be determined fairly precisely. The bat is in contact with the ball for an interval that is quite short compared with the time during which we are watching the ball. During the collision the bat exerts a large force on the ball (Fig. 6-1). This force varies with time in a complex way that we can measure only with difficulty. Both the ball and the bat are deformed during the collision. Forces that act for a time that is short compared with the time of observation of the system are called *impulsive* forces.

When an alpha particle (^{4}He nucleus) collides with another nucleus (Fig. 6-2), the force exerted on each by the other may be the repulsive electrostatic force associated with the charges on the particles. The particles may not actually come into direct contact with each other, but we still may speak of this interaction as a collision because a relatively

FIGURE 6-1. A high-speed photograph of a bat striking a baseball. Note the deformation of the ball, indicating the large impulsive force exerted by the bat.

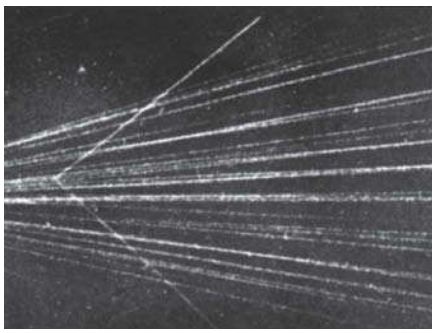


FIGURE 6-2. An alpha particle collides with a helium nucleus in a cloud chamber. Most of the incident particles (coming from the left) pass through without colliding.

strong force, acting for a time that is short compared with the time that the alpha particle is under observation, has a substantial effect on the motion of the alpha particle.

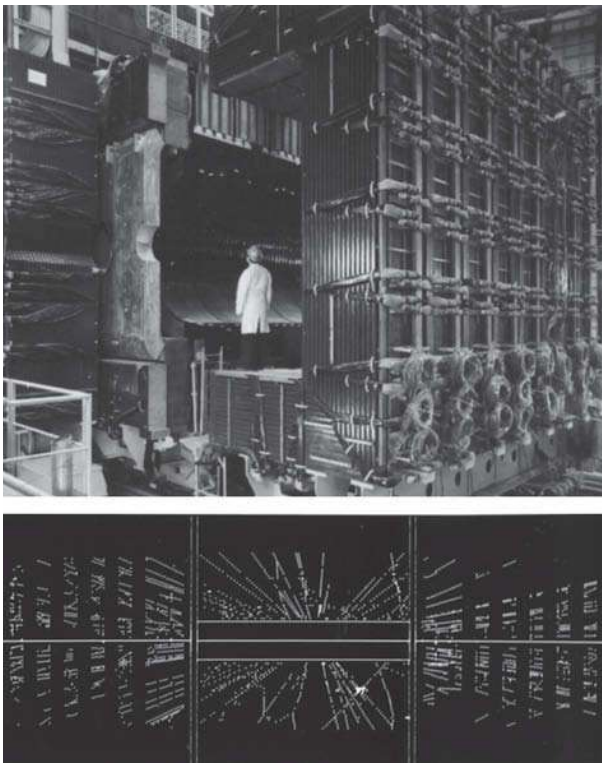
We can even speak about a collision between two galaxies (Fig. 6-3), if we are prepared to observe them over a time scale of the order of millions or billions of years. (However, a more feasible alternative is to shorten this long time span by computer modeling!)

Collisions between elementary particles provide the principal source of information about their internal structure. When two particles collide at high energy, often the products of the collision are very different from the original particles (Fig. 6-4). Sometimes these collisions produce hundreds of product particles. By studying the trajectories of the outgoing particles and applying fundamental laws, we can reconstruct the original event.

On a different scale, those who study traffic accidents also try to reconstruct collisions. From the paths and impact patterns of the colliding vehicles (Fig. 6-5), it is often possible to deduce such important details as the speeds and directions of motion of the two vehicles before the collision.

Another kind of collision is one that takes place between a space probe and a planet, called the "slingshot effect," in which the speed of the space probe can be increased in a "close encounter" with a (moving) planet. The probe does not actually touch the planet, but it does come strongly under its gravitational influence for a time that is very short compared to the duration of the space probe's

FIGURE 6-3. Two galaxies colliding.



(*a*)

(*b*)

FIGURE 6-4. (*a*) The massive detector UA1 used at the proton–proton collider at CERN, the particle physics research facility near Geneva, Switzerland. (*b*) A computer reconstruction of the paths of the particles produced in one proton–proton collision. Such reconstructions were used in 1983 to confirm the existence of the very short-lived particles called W and Z, which verified a theory that treats the electromagnetic force and the weak nuclear force as different aspects of a single more basic force.

journey. Thus we are justified in calling such encounters "collisions." For example, the Venus and Earth encounters of the Cassini mission (see Fig. 5-24) increased the spacecraft's speed by the equivalent of 75 tons of launch rocket fuel! Without this gravity assist, the Cassini spacecraft could not reach Saturn. (See Problem 14.)

In principle it would be possible to analyze each of these collisions using Newton's second law. Given the initial motion of each object and the force that acts between them, we could use the methods of Chapter 5 to find the velocity and position of each colliding object as a function of the time. However, there are two reasons why this is not possible for the collisions shown in Figs. 6-1 to 6-5: (1) For some of the collisions, we don't know the exact form of the expression for the force between the objects. (2) The colliding objects are composed of many particles, and it is hopelessly complicated to keep track of the application of Newton's laws for the force between each particle of one colliding object and each particle of the other.

Here is the basic problem: We have two objects with different initial motions that are originally so far apart that neither exerts a measurable force on the other. Eventually they

FIGURE 6-5. A collision between two automobiles. Momentum conservation is used by accident reconstruction experts to calculate the velocities before the collision.

approach one another, so that each exerts a force on the other and alters its motion. This force occurs for a time that is relatively short compared with the entire motion of the objects. Finally, they separate again with new motions and no longer interact. We observe the motion before the collision and the motion after the collision, but during the brief collision we do not observe or measure what is happening.

If we know the initial motions, can we find the final motions even though we do not know the force that acts to change the motions? Surprisingly, the answer is "yes!" In the next section, we define a new dynamic variable that enables us to analyze such collisions.

6-2 LINEAR MOMENTUM

To analyze collisions, we need a new dynamic variable, the *linear momentum* of a body. (Later we will introduce a similar variable for rotational motion, called the *angular* momentum. For the time being, we will refer to linear momentum simply as "momentum.") The momentum $\vec{\mathbf{p}}$ of a body is defined as the product of its mass and its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}. \tag{6-1}$$

As the product of a vector and a scalar, momentum must also be a vector. Equation 6-1 indicates that the direction of $\vec{\mathbf{p}}$ is the same as the direction of $\vec{\mathbf{v}}$. Because $\vec{\mathbf{p}}$ depends on $\vec{\mathbf{v}}$, *the momentum (like the velocity) depends on the reference frame of the observer, and we must always specify this frame.*

Newton, in his famous *Principia,* expressed the second law of motion in terms of momentum (which he called "quantity of motion"). Expressed in modern terminology Newton's second law reads:

> *The rate of change of momentum of a body is equal to the resultant force acting on the body and is in the direction of that force.*

In symbolic form this becomes

$$\sum \vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}. \tag{6-2}$$

Here $\Sigma\,\vec{\mathbf{F}}$ represents the resultant force acting on the particle.

For a single particle of constant mass, this form of the second law is equivalent to the form $\Sigma\,\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ that we have used up to now. That is, if m is constant, then

$$\sum \vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} = \frac{d}{dt}(m\vec{\mathbf{v}}) = m\frac{d\vec{\mathbf{v}}}{dt} = m\vec{\mathbf{a}}.$$

The relations $\Sigma\,\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ and $\Sigma\,\vec{\mathbf{F}} = d\vec{\mathbf{p}}/dt$ for single particles are completely equivalent in classical mechanics.

The equivalence of $\Sigma\,\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ and $\Sigma\,\vec{\mathbf{F}} = d\vec{\mathbf{p}}/dt$ depends, as you can see from the above equation, on the mass being a constant so that it passes through the derivative: $d(m\vec{\mathbf{v}})/dt = m(d\vec{\mathbf{v}}/dt)$. We shall assume that this is the case for the problems we discuss in this chapter. Section 7-6 covers applications of Newton's laws to systems in which the mass changes, such as a rocket that exhausts burning gases.

6-3 IMPULSE AND MOMENTUM

In this section we consider the relationship between the force that acts on a body during a collision and the change in the momentum of that body. During a collision, the force varies with time. For example, Fig. 6-6 shows how the magnitude of the force might change with time during a collision. The force is exerted only during the collision, which begins at time t_i and ends at time t_f. The force is zero before and after the collision.

From Newton's second law in the form of Eq. 6-2 ($\Sigma\,\vec{\mathbf{F}} = d\vec{\mathbf{p}}/dt$), we can write the change in momentum $d\vec{\mathbf{p}}$

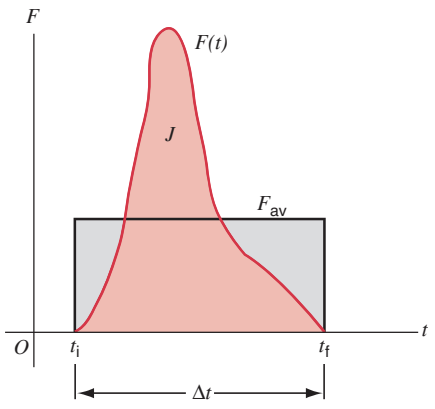


FIGURE 6-6. An impulsive force $F(t)$ varies in an arbitrary way with time during a collision that lasts from t_i to t_f. The area under the $F(t)$ curve is the impulse J, and the rectangle bounded by the average force F_{av} has an equal area.

of a particle in a time interval dt during which a net force $\Sigma \vec{\mathbf{F}}$ acts on it as

$$d\vec{\mathbf{p}} = \sum \vec{\mathbf{F}}\, dt.$$

To find the total change in momentum during the entire collision, we integrate over the time of collision, starting at time t_i (when the momentum is $\vec{\mathbf{p}}_i$) and ending at time t_f (when the momentum is $\vec{\mathbf{p}}_f$):

$$\int_{\vec{\mathbf{p}}_i}^{\vec{\mathbf{p}}_f} d\vec{\mathbf{p}} = \int_{t_i}^{t_f} \sum \vec{\mathbf{F}}\, dt. \tag{6-3}$$

The left side of Eq. 6-3 is the change in momentum, $\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$. The right side defines a new quantity called the *impulse*. For any arbitrary force $\vec{\mathbf{F}}$, the impulse $\vec{\mathbf{J}}$ is defined as

$$\vec{\mathbf{J}} = \int_{t_i}^{t_f} \vec{\mathbf{F}}\, dt. \tag{6-4}$$

The impulse depends on the strength of the force and on its duration. The impulse is a vector and, as Eq. 6-3 shows, the impulse has the same units and dimensions as momentum.

The right side of Eq. 6-3 is the impulse of the *net* force, $\vec{\mathbf{J}}_{net} = \int \Sigma\vec{\mathbf{F}}\, dt$. We can therefore write Eq. 6-3 as

$$\vec{\mathbf{J}}_{net} = \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i. \tag{6-5}$$

Equation 6-5 is the mathematical statement of the *impulse-momentum theorem:*

> *The impulse of the net force acting on a particle during a given time interval is equal to the change in momentum of the particle during that interval.*

As a vector relationship, Eq. 6-5 contains within it the three component equations:

$$\begin{aligned} J_{net,x} &= \Delta p_x = p_{fx} - p_{ix}, \\ J_{net,y} &= \Delta p_y = p_{fy} - p_{iy}, \\ J_{net,z} &= \Delta p_z = p_{fz} - p_{iz}. \end{aligned} \tag{6-6}$$

Although in this chapter we use Eq. 6-5 only in situations involving impulsive forces (that is, those of short duration compared with the time of observation), no such limitation is built into that equation. Equation 6-5 is just as general as Newton's second law, from which it was derived. We could, for example, use Eq. 6-5 to find the momentum acquired by a body falling in the Earth's gravity.

We defined the impulse in terms of a single force, but the impulse-momentum theorem deals with the change in momentum due to the impulse of the *net* force—that is, the combined effect of all the forces that act on the particle. In the case of a collision involving two particles, there is often no distinction because each particle is acted upon by only one force, which is due to the other particle. In this case, the change in momentum of one particle is equal to the impulse of the force exerted by the other particle.

The impulsive force whose magnitude is represented in Fig. 6-6 is assumed to have a constant direction. The magnitude of the impulse of this force is represented by the area under the $F(t)$ curve. We can represent that same area by the rectangle in Fig. 6-6 of width Δt and height F_{av}, where F_{av} is the magnitude of the *average force* that acts during the interval Δt. Thus

$$J = F_{av}\, \Delta t. \tag{6-7}$$

In a collision such as that of the ball and bat of Fig. 6-1, it is difficult to measure $F(t)$ directly, but we can estimate Δt (perhaps a few milliseconds) and obtain a reasonable value for F_{av} based on the impulse computed according to Eq. 6-6 from the change in momentum of the ball (see Sample Problem 6-1).

We have defined a collision as an interaction that occurs in a time Δt that is negligible compared to the time during which we are observing the system. We can also characterize a collision as an event in which the external forces that may act on the system during the time of the collision are negligible compared to the impulsive collision forces. While a bat strikes a baseball, a golf club strikes a golf ball, or one billiard ball strikes another, external forces act on the system. Gravity or friction may exert forces on these bodies, for example; these external forces may not be the same on each colliding body nor are they necessarily canceled by other external forces. Even so, it is quite safe to neglect these external forces during the collision. As a result, the change in momentum of a particle arising from an external force during a collision is negligible compared to the change in momentum of that particle arising from the impulsive collisional force (Fig. 6-7).

For example, when a bat strikes a baseball, the collision lasts only a few milliseconds. Because the change in momentum of the ball is large and the time of collision is small, it follows from

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{av}\, \Delta t \tag{6-8}$$

that the average impulsive force $\vec{\mathbf{F}}_{av}$ is relatively large. Compared to this force, the external force of gravity is neg-

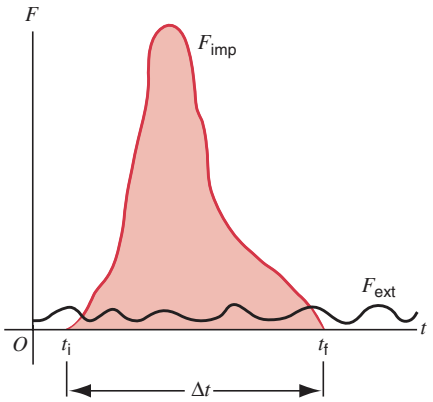


FIGURE 6-7. The impulsive force F_{imp} that acts during a collision is generally much stronger than any external force F_{ext} that may also act.

ligible. During the collision we can safely ignore this external force in determining the change in motion of the ball; the shorter the duration of the collision is, the more likely this is to be true.

SAMPLE PROBLEM 6-1. A baseball (which has an official weight of about 5 oz or a mass of 0.14 kg) is moving horizontally at a speed of 93 mi/h (about 42 m/s) when it is struck by the bat (see Fig. 6-1). It leaves the bat in a direction at an an angle $\phi = 35°$ above its incident path and with a speed of 50 m/s. (*a*) Find the impulse of the force exerted on the ball. (*b*) Assuming the collision lasts for 1.5 ms (= 0.0015 s), what is the average force? (*c*) Find the change in the momentum of the bat.

Solution (*a*) Figure 6-8*a* shows the initial momentum vector $\vec{\mathbf{p}}_i$ and the final momentum vector $\vec{\mathbf{p}}_f$ of the baseball. The components of the final momentum are given by

$$p_{fx} = mv_f \cos\phi = (0.14\text{ kg})(50\text{ m/s})(\cos 35°) = 5.7\text{ kg}\cdot\text{m/s},$$

$$p_{fy} = mv_f \sin\phi = (0.14\text{ kg})(50\text{ m/s})(\sin 35°) = 4.0\text{ kg}\cdot\text{m/s}.$$

In this coordinate system, the initial momentum has only an x component, whose (negative) value is

$$p_{ix} = mv_i = (0.14\text{ kg})(-42\text{ m/s}) = -5.9\text{ kg}\cdot\text{m/s}.$$

The impulse can now be found using Eq. 6-6:

$$J_x = p_{fx} - p_{ix} = 5.7\text{ kg}\cdot\text{m/s} - (-5.9\text{ kg}\cdot\text{m/s}) = 11.6\text{ kg}\cdot\text{m/s},$$

$$J_y = p_{fy} - p_{iy} = 4.0\text{ kg}\cdot\text{m/s} - 0 = 4.0\text{ kg}\cdot\text{m/s}.$$

In other terms, the impulse has magnitude

$$J = \sqrt{J_x^2 + J_y^2} = \sqrt{(11.6\text{ kg}\cdot\text{m/s})^2 + (4.0\text{ kg}\cdot\text{m/s})^2}$$
$$= 12.3\text{ kg}\cdot\text{m/s}$$

and acts in a direction determined by

$$\theta = \tan^{-1}(J_y/J_x) = \tan^{-1}[(4.0\text{ kg}\cdot\text{m/s})/(11.6\text{ kg}\cdot\text{m/s})] = 19°$$

above the horizontal. Figure 6-8*b* shows the impulse vector $\vec{\mathbf{J}}$ and verifies graphically that, as the definition of Eq. 6-6 requires,

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f + (-\vec{\mathbf{p}}_i).$$

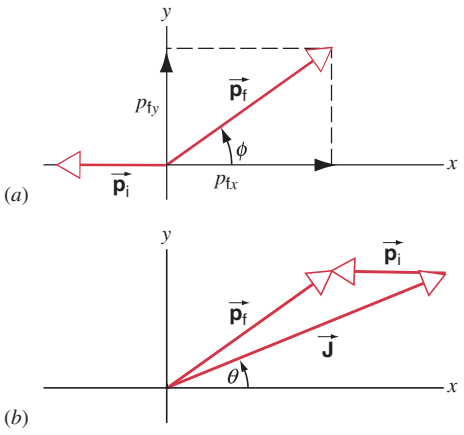


FIGURE 6-8. Sample Problem 6-1. (*a*) The initial and final momenta of the baseball. (*b*) The difference $\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$ is equal to the impulse $\vec{\mathbf{J}}$.

(*b*) Using Eq. 6-7, we obtain

$$F_{av} = J/\Delta t = (12.3\text{ kg}\cdot\text{m/s})/0.0015\text{ s} = 8200\text{ N},$$

which is nearly 1 ton. This force acts in the same direction as $\vec{\mathbf{J}}$—that is, 19° above the horizontal. Note that this is the *average* force; the *maximum* force is considerably greater, as Fig. 6-6 shows. Also, note that F_{av} (= 8200 N) $\gg mg$ (= 1.4 N). Thus we are quite safe in assuming that the impulsive force greatly exceeds the external force (gravity, in this case) and therefore is very nearly equal to the net force that acts during the collision.

(*c*) From Newton's third law, the force exerted on the bat by the ball is equal and opposite to the force exerted on the ball by the bat. Therefore, according to Eq. 6-8, the change in momentum of the bat is equal and opposite to that of the ball. Thus, for the bat,

$$\Delta p_x = -11.6\text{ kg}\cdot\text{m/s},$$

$$\Delta p_y = -4.0\text{ kg}\cdot\text{m/s}.$$

Is this a large change or a small one? Try to estimate the momentum of the bat in motion to answer this question.

SAMPLE PROBLEM 6-2. A cart of mass $m_1 = 0.24$ kg moves on a linear track without friction with an initial velocity of 0.17 m/s. It collides with another cart of mass $m_2 = 0.68$ kg that is initially at rest. The first cart carries a force probe that registers the magnitude of the force exerted by one cart on the other during the collision. The output of the force probe is shown in Fig. 6-9. Find the velocity of each cart after the collision.

Solution Our strategy in this problem is to find the impulse from the force graph. The impulse gives the change in momentum, which allows us to find the final momentum of each cart. The impulse $\int F\,dt$ is the area under the graph of $F(t)$ in Fig. 6-9, which can be determined as the area of a triangle:

$$J = \int F\,dt = \tfrac{1}{2}(0.014\text{ s} - 0.003\text{ s})(10\text{ N})$$
$$= 0.055\text{ N}\cdot\text{s} = 0.055\text{ kg}\cdot\text{m/s}.$$

Since the graph gives the magnitude of the force, this integral gives the magnitude of the impulse. We take the direction of motion of the first cart as the positive x direction. Then the x component force exerted on cart 1 by cart 2 is negative, and so the corresponding component of $\vec{\mathbf{J}}$ is negative. Since $J_x = \Delta p_x$, for the first

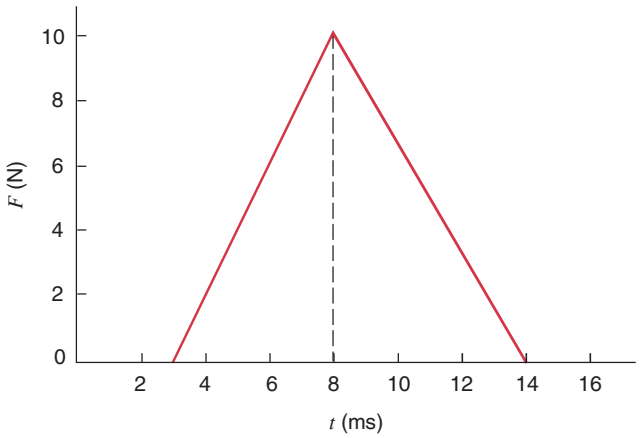


FIGURE 6-9. Sample Problem 6-2.

cart we have $\Delta p_{1x} = -0.055\ \text{kg}\cdot\text{m/s}$ and so its final momentum and velocity are

$$p_{1fx} = p_{1ix} + \Delta p_{1x} = (0.24\ \text{kg})(0.17\ \text{m/s}) - 0.055\ \text{kg}\cdot\text{m/s}$$
$$= -0.014\ \text{kg}\cdot\text{m/s}$$

$$v_{1fx} = \frac{p_{1fx}}{m_1} = \frac{-0.014\ \text{kg}\cdot\text{m/s}}{0.24\ \text{kg}} = -0.058\ \text{m/s} = -5.8\ \text{cm/s}.$$

Cart 1 moves in the negative x direction after the collision.

The force on cart 2 is, by Newton's third law, equal and opposite to the force on cart 1, so it is in the positive x direction. Because the forces are equal in magnitude, the impulses are equal in magnitude but opposite in direction. So $J_{2x} = \Delta p_{2x} = +0.055\ \text{kg}\cdot\text{m/s}$, and the final momentum and velocity of cart 2 are

$$p_{2fx} = p_{2ix} + \Delta p_{2x} = 0 + 0.055\ \text{kg}\cdot\text{m/s} = +0.055\ \text{kg}\cdot\text{m/s}$$

$$v_{2fx} = \frac{p_{2fx}}{m_2} = \frac{+0.055\ \text{kg}\cdot\text{m/s}}{0.68\ \text{kg}} = +0.081\ \text{m/s} = +8.1\ \text{cm/s}.$$

Cart 2 moves in the positive x direction after the collision.

6-4 CONSERVATION OF MOMENTUM

In this section we consider the analysis of collisions between two objects, each of which may be moving. In contrast to Sample Problem 6-2, the objects may be moving in any direction, so we must use vectors to describe the motion.

Figure 6-10*a* illustrates the general problem. A body with mass m_1 moves initially with velocity $\vec{\mathbf{v}}_{1i}$ and momentum $\vec{\mathbf{p}}_{1i} = m_1\vec{\mathbf{v}}_{1i}$. It collides with body 2, which is moving initially with velocity $\vec{\mathbf{v}}_{2i}$ and momentum $\vec{\mathbf{p}}_{2i} = m_2\vec{\mathbf{v}}_{2i}$. We focus our attention on the motions of the two bodies, which we define as our *system*. We assume that the system consisting of the two bodies is isolated from its environment, so that no forces act on either body during the collision except the impulsive force that each body exerts on the other. After the collision (Fig. 6-10*b*), m_1 moves with velocity $\vec{\mathbf{v}}_{1f}$ and momentum $\vec{\mathbf{p}}_{1f} = m_1\vec{\mathbf{v}}_{1f}$ and m_2 with velocity $\vec{\mathbf{v}}_{2f}$ and momentum $\vec{\mathbf{p}}_{2f} = m_2\vec{\mathbf{v}}_{2f}$.

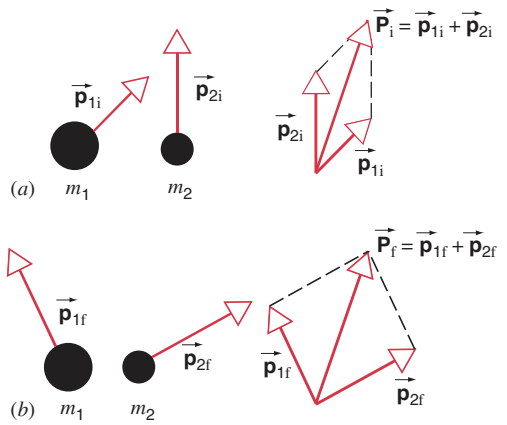


FIGURE 6-10. (*a*) Two objects and their momenta before they collide. (*b*) The objects and their momenta after they collide. Note that the total momentum vectors $\vec{\mathbf{P}}_i$ and $\vec{\mathbf{P}}_f$ are the same before and after the collision.

At any particular time, the total momentum of the system consisting of the two bodies is

$$\vec{\mathbf{P}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2, \tag{6-9}$$

which we can evaluate before, during, or after the collision. Taking the time derivative of Eq. 6-9, we obtain

$$\frac{d\vec{\mathbf{P}}}{dt} = \frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} = \sum \vec{\mathbf{F}}_1 + \sum \vec{\mathbf{F}}_2, \tag{6-10}$$

where we have used Eq. 6-2 to replace $d\vec{\mathbf{p}}/dt$ for each body with the net force acting on that body. Before the collision, no forces act on the bodies, so $\Sigma\,\vec{\mathbf{F}}_1 = 0$ and $\Sigma\,\vec{\mathbf{F}}_2 = 0$, and therefore $d\vec{\mathbf{P}}/dt = 0$. Similarly, after the collision $d\vec{\mathbf{P}}/dt = 0$ because again no forces act on the bodies. During the collision, the only force acting on body 1 is $\vec{\mathbf{F}}_{12}$, which is due to body 2. Similarly, $\vec{\mathbf{F}}_{21}$ is the only force acting on body 2 during the collision. $\vec{\mathbf{F}}_{12}$ and $\vec{\mathbf{F}}_{21}$ form an action–reaction pair, so $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$ and $\vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{21} = 0$. Thus $d\vec{\mathbf{P}}/dt = 0$ during the collision, too. So we get the same result when we evaluate Eq. 6-10 before, during, and after the collision: at all times,

$$\frac{d\vec{\mathbf{P}}}{dt} = 0. \tag{6-11}$$

If the time derivative of a quantity is zero, then that quantity does not change with time and must be a constant:

$$\vec{\mathbf{P}} = \text{constant}. \tag{6-12}$$

That is, the total momentum of m_1 and m_2 before the collision must be the same in magnitude and direction as the total momentum of m_1 and m_2 after the collision. Even though $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ may both change as a result of the collision, their vector sum stays the same (as in Fig. 6-10).

Another way of expressing this result is

$$\vec{\mathbf{P}}_i = \vec{\mathbf{P}}_f, \tag{6-13}$$

where $\vec{\mathbf{P}}_i = \vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i}$ is the total initial momentum of the system before the collision and $\vec{\mathbf{P}}_f = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$ is the total final momentum after the collision.

Equations 6-11, 6-12, and 6-13 are equivalent mathematical statements of the *law of conservation of linear momentum* for an isolated system consisting of two bodies:

> *When the net external force acting on a system is zero, the total linear momentum of the system remains constant.*

This is a general result, valid for any type of interaction between the bodies. It is not even necessary that the bodies behave like particles for this law to be valid (as in the collision of Fig. 6-5). Although we obtained this result for a two-body system, the law of conservation of momentum is perfectly general and applies to any collection or system of

bodies in which the only forces that act are those that the bodies in the system exert on each other.

Momentum is a vector quantity so for momentum to be conserved all three components must be conserved independently. For example, Eq. 6-12 gives

$$P_x = \text{constant}, \qquad P_y = \text{constant}, \qquad P_z = \text{constant}. \tag{6-14}$$

The total x component of the momentum stays the same before and after the collision, as do the y and z components.

Because we derived the law of conservation of momentum using Newton's laws, the law is valid in any inertial frame of reference. Observers in different inertial frames watching the same collision may disagree on the values they measure for the initial and final momenta, but each will agree that the initial and final momenta are equal. If momentum is conserved in one inertial frame, it is conserved in *every* inertial frame.

Conservation laws have an important role in our analysis and understanding of physical processes. They allow us to compare the behavior of a system "before" and "after," without having any detailed knowledge of the processes that happen "in between." For example, the law of conservation of linear momentum makes no assumptions about the type of force that the two bodies exert on one another; the total linear momentum before the collision will equal the total momentum after the collision, no matter what type of force acts on the colliding objects.

Later in this text we shall encounter other conservation laws, including those for energy, angular momentum, and electric charge. These laws are of great practical and theoretical importance. There is a deep theoretical connection between conserved quantities and symmetries of nature. For instance, the law of conservation of linear momentum is connected to the *spatial* symmetry of nature, which requires that an experiment done at one location should yield the identical result as the same experiment done at another location. Later we will discuss another conservation law, the conservation of energy, which is connected to *temporal* (time) symmetry: the result of an experiment done today should agree with the result of the same experiment done yesterday. Because of these connections, we believe that these two conservation laws are universally valid—if the nature of space and time is the same everywhere in the universe, then the same conservation laws should apply everywhere and at all times.

SAMPLE PROBLEM 6-3. Fred ($m_F = 75$ kg) and Ginger ($m_G = 55$ kg) are ice skating side by side at a common velocity of 3.2 m/s (Fig. 6-11*a*) when they push off from one another in a direction perpendicular to their original velocity. After they break contact, Ginger is skating in a direction at an angle of 32° from her original direction (Fig. 6-11*b*). In what direction is Fred now skating?

Solution We take Fred and Ginger together to be our system. Fred and Ginger exert forces on each other when they push apart,

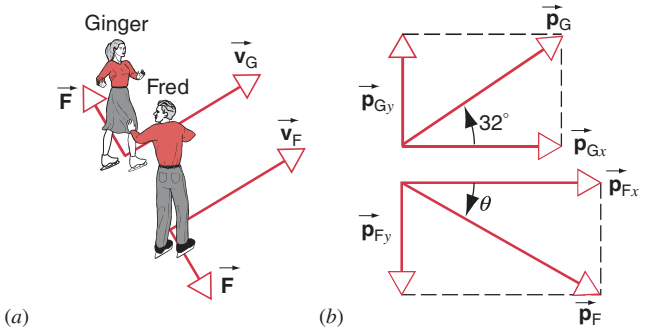


FIGURE 6-11. Sample Problem 6-3. (*a*) Two skaters push off from one another in a direction perpendicular to their original motion. (*b*) The momenta of the skaters after they push off.

but if we neglect the effect of any external forces (such as friction with the ice), their total momentum before they push apart must be the same as their total momentum after they push apart. The x components of their original momenta are (taking their original motion to be in the positive x direction):

$$p_{Gx} = m_G v_G = (55 \text{ kg})(3.2 \text{ m/s}) = 176 \text{ kg}\cdot\text{m/s}$$

$$p_{Fx} = m_F v_F = (75 \text{ kg})(3.2 \text{ m/s}) = 240 \text{ kg}\cdot\text{m/s}.$$

After they push off, Ginger's momentum acquires a y component so that her total momentum makes an angle of 32° with the positive x direction:

$$p_{Gy} = p_{Gx} \tan 32° = +110 \text{ kg}\cdot\text{m/s}.$$

Before they push apart, the y component of their total momentum is zero. For momentum to be conserved, the total y component must remain zero after they separate. Therefore, the y component of Fred's momentum must be -110 kg·m/s, and the direction of Fred's motion is determined from

$$\tan\theta = \frac{p_{Fy}}{p_{Fx}} = \frac{-110 \text{ kg}\cdot\text{m/s}}{240 \text{ kg}\cdot\text{m/s}} = -0.458$$

or

$$\theta = -25°.$$

Note that the x component of the momentum of either Fred or Ginger is unchanged by the force of their pushing away from each other; this force is exerted in the y direction, and the impulse-momentum theorem (Eq. 6-6) tells us that a force on either of them in the y direction cannot change the x component of their momentum.

SAMPLE PROBLEM 6-4. A man of mass 65 kg is running along a pier at a speed of 4.9 m/s (Fig. 6-12). He jumps from the pier into a rowboat of mass 88 kg that is drifting without friction in the same direction at a speed of 1.2 m/s. When the man is seated in the rowboat, what is its final velocity?

Solution As the man enters the boat, he and the boat exert forces on each other that cause them to acquire the same final speed (the man slows down and the boat speeds up). If there are no external forces acting on the system of man + boat, the total momentum of the man and the boat before he jumps must equal the total momentum after he is seated in the boat. We choose the positive x

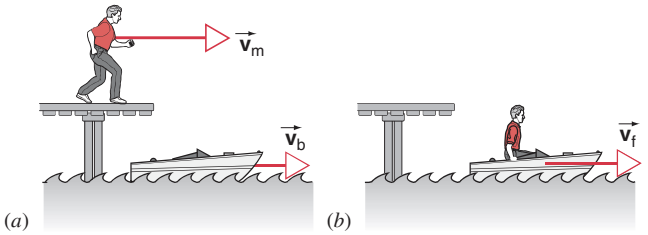


FIGURE 6-12. Sample Problem 6-4. (*a*) A man runs with velocity $\vec{\mathbf{v}}_m$ and jumps into a boat moving in the same direction with velocity $\vec{\mathbf{v}}_b$. (*b*) The man and boat are together moving with velocity $\vec{\mathbf{v}}_f$.

axis in the direction of the man's original velocity, and because all motion is in the x direction we need consider only the x components of all velocities and momenta. Before he jumps, the man has momentum $p_{mx} = m_m v_{mx}$ and the boat has momentum $p_{bx} = m_b v_{bx}$. The total initial momentum is

$$P_{ix} = p_{mx} + p_{bx} = m_m v_{mx} + m_b v_{bx}$$
$$= (65\text{ kg})(4.9\text{ m/s}) + (88\text{ kg})(1.2\text{ m/s}) = 424\text{ kg}\cdot\text{m/s}.$$

After he jumps and is seated in the boat, they move together with the same velocity v_{fx}. Their combined final momentum is $P_{fx} = m_m v_{fx} + m_b v_{fx} = (m_m + m_b)v_{fx}$. With $P_{ix} = P_{fx}$, we obtain

$$v_{fx} = \frac{P_{ix}}{m_m + m_b} = \frac{424\text{ kg}\cdot\text{m/s}}{65\text{ kg} + 88\text{ kg}} = 2.8\text{ m/s}.$$

6-5 TWO-BODY COLLISIONS

In this section we examine different types of two-body collisions using momentum conservation to relate the motion of the bodies before and after the collision.

Figure 6-13*a* shows a general two-body collision. Before the collision, m_1 moves with initial velocity $\vec{\mathbf{v}}_{1i}$ and m_2 with initial velocity $\vec{\mathbf{v}}_{2i}$. After the collision, the final velocities are $\vec{\mathbf{v}}_{1f}$ and $\vec{\mathbf{v}}_{2f}$ respectively. According to conservation of momentum, the total momentum of m_1 and m_2 *before* the collision equals their total momentum *after* the collision:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}. \qquad (6\text{-}15)$$

Another way of writing Eq. 6-15 is

$$m_1(\vec{\mathbf{v}}_{1f} - \vec{\mathbf{v}}_{1i}) = -m_2(\vec{\mathbf{v}}_{2f} - \vec{\mathbf{v}}_{2i}) \qquad (6\text{-}16)$$

or

$$\Delta\vec{\mathbf{p}}_1 = -\Delta\vec{\mathbf{p}}_2. \qquad (6\text{-}17)$$

The changes in momentum of the two objects have equal magnitudes and opposite signs, a necessary consequence of the law of conservation of momentum. This result also follows directly from Newton's third law: according to the impulse-momentum theorem (Eq. 6-5), the change in momentum of either body equals the impulse of the net force that acts on that body. Equation 6-17 can thus be written as $\vec{\mathbf{J}}_1 = -\vec{\mathbf{J}}_2$ where $\vec{\mathbf{J}}_1$ means the impulse of the force on body 1 due to body 2 and $\vec{\mathbf{J}}_2$ means the impulse of the force on body 2 due to body 1. This equality follows directly from the definition of the impulse (Eq. 6-4) with $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$, as required by Newton's third law.

In some collisions, the bodies stick together (Fig. 6-13*b*) and move with a common final velocity. With $\vec{\mathbf{v}}_{1f} = \vec{\mathbf{v}}_{2f} = \vec{\mathbf{v}}_f$, Eq. 6-15 becomes

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = (m_1 + m_2)\vec{\mathbf{v}}_f. \qquad (6\text{-}18)$$

Equations 6-15 and 6-18 are vector equations, which implies that conservation of momentum must be valid for each of the components, as suggested by Eq. 6-14. Thus

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

and similarly for the y and z components. If all motion takes place in a plane (the xy plane) and if we know the initial velocities of m_1 and m_2, then Eq. 6-15 gives two relationships among the four unknowns (the x and y components of $\vec{\mathbf{v}}_{1f}$ and $\vec{\mathbf{v}}_{2f}$). If we also know *one* of the final velocities, we can find the other; or if we know the *directions* of the two final velocities, we can find their magnitudes. Equation 6-18, on the other hand, has only two unknowns (the x and y components of $\vec{\mathbf{v}}_f$), so the two component equations contained in Eq. 6-18 are therefore sufficient to solve for these two unknowns.

In many applications, m_2 is initially at rest ($\vec{\mathbf{v}}_{2i} = 0$). This simplifies the calculation somewhat. Since momentum conservation is valid in any inertial frame, we can always find a frame of reference in which m_2 is at rest and apply momentum conservation in that frame, returning to the original frame of reference if we wish to evaluate the final velocities in that frame.

Often we have a "head-on" collision in which all of the motion occurs only in one direction, which we take to be

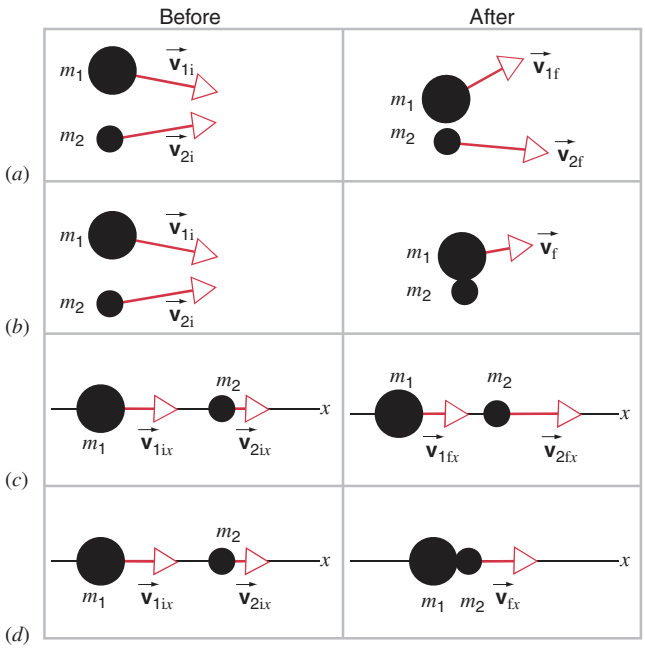


FIGURE 6-13. The initial and final velocities in various two-body collisions.

the x direction of our coordinate system (Fig. 6-13c). Conservation of momentum in this case can be written

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}. \qquad (6\text{-}19)$$

If three of the velocities are known, the fourth can be found from Eq. 6-19. If m_1 and m_2 stick together after the collision and move with a common final velocity v_{fx} (Fig. 6-13d), Eq. 6-19 becomes

$$m_1 v_{1ix} + m_2 v_{2ix} = (m_1 + m_2) v_{fx}. \qquad (6\text{-}20)$$

The x components of the velocities in Eqs. 6-19 and 6-20 can be positive or negative, depending on how we define the positive direction of the x axis.

SAMPLE PROBLEM 6-5. A glider of mass $m_1 = 1.25$ kg moves with a velocity of 3.62 m/s on a frictionless, level air track and collides with a second glider of mass $m_2 = 2.30$ kg that is initially at rest. After the collision, the first glider is found to be moving at 1.07 m/s in a direction opposite to that of its initial motion. What is the velocity of m_2 after the collision?

Solution Equation 6-19 gives the general momentum conservation result in one dimension. We choose the positive x direction to be that of the initial motion of m_1, so that $v_{1ix} = +3.62$ m/s and $v_{1fx} = -1.07$ m/s. With $v_{2ix} = 0$, we can solve Eq. 6-19 for the unknown v_{2fx} and obtain

$$v_{2fx} = \frac{m_1}{m_2}(v_{1ix} - v_{1fx})$$

$$= \frac{1.25\text{ kg}}{2.30\text{ kg}}[3.62\text{ m/s} - (-1.07\text{ m/s})] = 2.55\text{ m/s}.$$

We can check this result by finding the change in momentum for each of the gliders:

$$\Delta p_{1x} = m_1(v_{1fx} - v_{1ix}) = (1.25\text{ kg})(-1.07\text{ m/s} - 3.62\text{ m/s})$$
$$= -5.86\text{ kg}\cdot\text{m/s},$$

$$\Delta p_{2x} = m_2(v_{2fx} - v_{2ix}) = (2.30\text{ kg})(2.55\text{ m/s} - 0)$$
$$= +5.86\text{ kg}\cdot\text{m/s}.$$

As expected, $\Delta p_{1x} = -\Delta p_{2x}$.

SAMPLE PROBLEM 6-6. Suppose the two gliders moving initially as in Sample Problem 6-5 stick together after the collision. What is the final velocity of the combination?

Solution In this case we can use Eq. 6-20 with $v_{2ix} = 0$:

$$v_{fx} = \frac{m_1 v_{1ix}}{m_1 + m_2} = \frac{(1.25\text{ kg})(3.62\text{ m/s})}{1.25\text{ kg} + 2.30\text{ kg}} = 1.27\text{ m/s}.$$

Following the same method as in the previous Sample Problem, you should show that $\Delta p_{1x} = -2.93$ kg·m/s and $\Delta p_{2x} = +2.93$ kg·m/s, thereby satisfying conservation of momentum ($\Delta p_{1x} = -\Delta p_{2x}$).

SAMPLE PROBLEM 6-7. A spaceship of total mass m is coasting at a speed of 8.45 km/s (measured with respect to a particular inertial reference frame) in a region of space of negligible gravity. The ship consists of two craft of masses $m/4$ and $3m/4$ that are clamped together with a spring between them (Fig. 6-14). Upon a signal from the ship's commander, the bolts holding the two craft together are released, and the spring drives them apart such that the smaller craft moves forward (in the direction of the ship's original motion) at a speed of 11.63 km/s. What is the final speed of the larger craft?

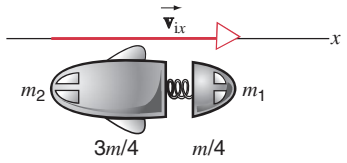


FIGURE 6-14. Sample Problem 6-7.

Solution This problem is the reverse of the previous one—instead of the two bodies colliding and sticking together, we have two bodies initially stuck together that come apart. The force that drives them apart is an internal force in the two-body system, so the law of conservation of momentum can be applied. We choose the positive x direction to be that of the original motion of the spaceship (and also that of the final velocity of the smaller craft). Let us turn Eq. 6-20 around, so that the velocity of the initial combined system is v_{ix} and the final velocities of the two pieces are v_{1fx} and v_{2fx}. Then momentum conservation gives

$$(m_1 + m_2) v_{ix} = m_1 v_{1fx} + m_2 v_{2fx}. \qquad (6\text{-}21)$$

We are given $v_{ix} = +8.45$ km/s and $v_{1fx} = +11.63$ km/s, and we wish to find v_{2fx}:

$$v_{2fx} = \frac{(m_1 + m_2)v_{ix} - m_1 v_{1fx}}{m_2}$$

$$= \frac{(m)(8.45\text{ km/s}) - (m/4)(11.63\text{ km/s})}{3m/4} = +7.39\text{ km/s}.$$

Note that we do not need to know the actual mass of the ship for this calculation, only the relative masses of the two pieces. Is this true for all two-body collisions? Review Eqs. 6-19 and 6-20 to decide whether this is true in general.

It is instructive to analyze this problem from the perspective of another spaceship that is moving parallel to the first at the same velocity ($v_x = 8.45$ km/s). Relative to this ship, the smaller craft after its release moves with a velocity of $v'_{1fx} = v_{1fx} - v_x = 11.63\text{ km/s} - 8.45\text{ km/s} = 3.18$ km/s in the forward direction (the same direction as the ship's velocity). The larger craft moves with a velocity of $v'_{2fx} = v_{2fx} - v_x = 7.39\text{ km/s} - 8.45\text{ km/s} = -1.06$ km/s. In this frame of reference, the larger craft moves backward with a speed of 1.06 km/s. According to this observer, the initial momentum of the first spaceship (before the separation) is zero, because the relative velocity of the first ship is zero. After the separation, the total final momentum of the two craft is zero: $P'_{fx} = m_1 v'_{1fx} + m_2 v'_{2fx} = 0$, as you can show. According to this observer, the initial and final momenta are both zero, and thus momentum is conserved.

If momentum is conserved in one inertial reference frame, then it is conserved in every inertial reference frame. Often it is convenient to solve a problem in one reference frame and then transform the results to another. In the remainder of this section we discuss how the second reference frame used in this problem, in which the total momentum is zero, can often yield insight into the analysis of collisions.

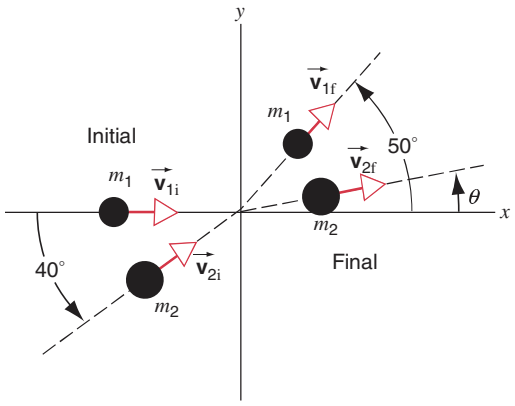


FIGURE 6-15. Sample Problem 6-8.

SAMPLE PROBLEM 6-8. A puck is sliding without friction on the ice at a speed of 2.48 m/s. It collides with a second puck of mass 1.5 times that of the first and moving initially with a velocity of 1.86 m/s in a direction 40° away from the direction of the first puck (Fig. 6-15). After the collision, the first puck moves at a velocity of 1.59 m/s in a direction at an angle of 50° from its initial direction (as shown in Fig. 6-15). Find the speed and direction of the second puck after the collision.

Solution In this problem we must use the law of conservation of momentum in its two-dimensional vector form. We define the x axis as the direction of the initial motion of the first puck. Let the second puck move with velocity $\vec{\mathbf{v}}_{2f}$ at an angle θ with the x axis. Then the x component of the conservation of momentum equation (Eq. 6-15) gives $m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$, or (with $m_1 = m$ and $m_2 = 1.5m$)

$$m(2.48\text{ m/s}) + 1.5m(1.86\text{ m/s})\cos 40° = m(1.59\text{ m/s})\cos 50° + 1.5mv_{2f}\cos\theta,$$

which reduces to

$$v_{2f}\cos\theta = 2.40\text{ m/s},$$

and the y component is $m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$, so

$$m(0) + 1.5m(1.86\text{ m/s})\sin 40° = m(1.59\text{ m/s})\sin 50° + 1.5mv_{2f}\sin\theta,$$

which reduces to

$$v_{2f}\sin\theta = 0.38\text{ m/s}.$$

Solving the two reduced equations for the two unknowns, we find

$$v_{2f} = 2.43\text{ m/s}, \qquad \theta = 9.0°.$$

One-Dimensional Collisions in the Center-of-Mass Reference Frame

Earlier in this section we analyzed a one-dimensional collision between two bodies, viewed from an arbitrary inertial reference frame. Often we choose that frame to be fixed in the laboratory in which the collision is observed, so it is called the *laboratory reference frame* or lab frame. There is, however, another special reference frame in which the collision displays a particular symmetry and in which the analysis can often be carried out with relative ease. This frame, which is widely adopted by physicists for analyzing collisions of atoms and subatomic particles, is called the *center-of-mass reference frame* or cm frame. (The reason for this choice of name will be explained in the next chapter.) In Sample Problem 6-7, the second method of analysis was carried out in the cm frame, in which the spaceship appeared to be at rest before the separation.

The (one-dimensional) collision in the lab frame is shown in Fig. 6-16*a*. We now analyze that collision in the cm frame. All velocities are in the x direction, so for convenience we drop the x subscript from the velocities and momenta; however, it must be remembered that these are x components of vectors and so their signs must be chosen consistently relative to the direction we defined as the positive x direction. We use Eq. 4-32 in one dimension, and we let S represent the lab frame and S' represent the cm frame. The velocity $v_{S'S}$ (the velocity of the cm frame relative to the lab frame) is represented simply as v. According to an observer in the cm frame, the initial velocities of the two colliding objects are

$$m_1: \quad v'_{1i} = v_{1i} - v \qquad \text{and} \qquad m_2: \quad v'_{2i} = v_{2i} - v,$$

where the primes represent quantities measured in the cm (S') frame.

The total initial momentum of the two bodies in the cm frame is then

$$P'_i = m_1v'_{1i} + m_2v'_{2i} = m_1(v_{1i} - v) + m_2(v_{2i} - v). \qquad (6\text{-}22)$$

We now *define* the cm frame to be *the frame in which the initial momentum of the two-body system is zero.* To find the particular value of v that will bring this about, we set $P'_i = 0$ in Eq. 6-22 and solve for v, obtaining

$$v = \frac{m_1v_{1i} + m_2v_{2i}}{m_1 + m_2}. \qquad (6\text{-}23)$$

If we travel at this velocity and observe the collision, the motion of the two bodies *before* the collision would appear as in Fig. 6-16*b*. Even though their masses may be different, the momenta of the two bodies are equal and opposite, so that their total is zero.

Because momentum is conserved, the total momentum *after* the collision ($P'_f = p'_{1f} + p'_{2f}$) must also be zero in the cm frame, so after the collision p'_{1f} and p'_{2f} must also be equal

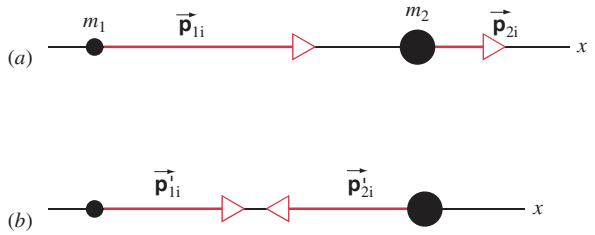


FIGURE 6-16. The momenta of two bodies before their collision in (*a*) the original frame of reference and (*b*) the center-of-mass frame of reference.

and opposite. The final momentum vectors may have any length, as long as they are equal to one another in magnitude.

Line 1 of Fig. 6-17, which is identical to Fig. 6-16*b*, shows the initial momenta of the two bodies in the cm frame. The outcome of the collision depends on the properties of the colliding bodies and on the nature of the forces they exert on each other; lines 2–5 of Fig. 6-17 show several different possibilities for the final momenta p'_{1f} and p'_{2f} in the cm frame. The total final momentum is zero in this frame, so that $p'_{1f} = -p'_{2f}$. Regardless of the type of collision, this symmetry is revealed in the cm frame.

In the case shown in line 2 of Fig. 6-17, the bodies simply bounce off one another, with their momenta unchanged in magnitude and reversed in direction. This type of collision is known as an *elastic* collision. Rigid solid objects, such as billiard balls or hockey pucks, usually experience collisions that can be regarded as elastic. In *inelastic* collisions (line 3), the bodies rebound with smaller momenta in the cm frame. This is the case for nonrigid objects, such as the baseball in Fig. 6-1. If the bodies stick together after the collision (line 4), the combination will be at rest in the cm frame; we call this a *completely inelastic* collision. The collision of two putty balls is an example. Finally (line 5), the bodies might rebound with momenta larger than their initial values. This might occur, for example, if a coiled spring or an explosive charge were released between the bodies at the instant of collision.

Elastic Collisions. We have defined an elastic collision as one in which, *in the cm frame,* the velocity of each body changes in direction but not in magnitude. Thus for m_1, $v'_{1f} = -v'_{1i}$ in the cm frame, and similarly for m_2. Now we use these results to derive expressions for the final velocities of the two elastically colliding bodies in the lab frame.

For m_1, the velocities in the two frames are related by $v'_{1i} = v_{1i} - v$ and $v'_{1f} = v_{1f} - v$, where v is the relative velocity between the frames (Eq. 6-23). Solving the latter of these two equations for the velocity in the lab frame, we obtain $v_{1f} = v'_{1f} + v$. Substituting the condition for elastic collisions ($v'_{1f} = -v'_{1i}$) then gives $v_{1f} = -v'_{1i} + v$. Finally, using the relationship between v'_{1i} and v_{1i}, we obtain

$$v_{1f} = -(v_{1i} - v) + v = -v_{1i} + 2v$$
$$= -v_{1i} + 2\frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2},$$

where the latter result comes from using Eq. 6-23. After some rearrangement, we get

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}. \qquad (6\text{-}24)$$

To find v_{2f}, the final velocity of m_2, we could repeat the analysis that led to Eq. 6-24, interchanging the subscripts "1" and "2" everywhere they appear. In fact, we can simply make those changes to Eq. 6-24, which gives

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}. \qquad (6\text{-}25)$$

Equations 6-24 and 6-25 are general results for one-dimensional elastic collisions and allow us to calculate the final velocities in *any* inertial reference frame in terms of the initial velocities in that frame. Here are some special cases of interest:

1. *Equal masses.* When the colliding particles have equal masses ($m_1 = m_2$), Eqs. 6-24 and 6-25 become simply

$$v_{1f} = v_{2i} \quad \text{and} \quad v_{2f} = v_{1i}. \qquad (6\text{-}26)$$

That is, the particles exchange velocities: the final velocity of one particle is equal to the initial velocity of the other.

2. *Target particle at rest.* Another case of interest is that in which particle m_2 is initially at rest. Then $v_{2i} = 0$ and

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \quad \text{and} \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}. \qquad (6\text{-}27)$$

Combining this special case with the previous one (that is, a collision between equal mass particles in which one is initially at rest), we see that the first particle is "stopped cold" and the second one "takes off" with the velocity the first one originally had. It is often possible to observe this effect in collisions of nonrotating billiard balls.

3. *Massive target.* If $m_2 >> m_1$, then Eqs. 6-24 and 6-25 reduce to

$$v_{1f} \approx -v_{1i} + 2v_{2i} \quad \text{and} \quad v_{2f} \approx v_{2i}. \qquad (6\text{-}28)$$

When the massive particle is moving slowly or at rest, then

$$v_{1f} \approx -v_{1i} \quad \text{and} \quad v_{2f} \approx 0. \qquad (6\text{-}29)$$

That is, when a light projectile collides with a very much more massive one at rest, the velocity of the light particle is approximately reversed, and the massive particle remains

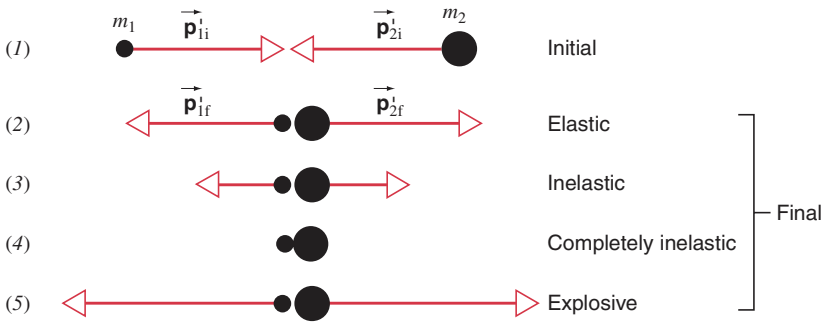


FIGURE 6-17. The momenta of two colliding objects in the center-of-mass frame for various types of collisions. Line 1 shows the initial momenta in this frame, and lines 2–5 show some possible final momenta.

approximately at rest. For example, a ball dropped from a height h rebounds from the Earth after the collision with reversed velocity and, if the collision were perfectly elastic and there were no air resistance, it would reach the same height h. Similarly, an electron rebounds from a (relatively massive) atom in a head-on collision with its motion reversed, while the target atom is essentially unaffected by the collision.

4. *Massive projectile.* When $m_1 >> m_2$, Eqs. 6-24 and 6-25 become

$$v_{1f} \approx v_{1i} \qquad \text{and} \qquad v_{2f} \approx 2v_{1i} - v_{2i}. \qquad (6\text{-}30)$$

If the light target particle is initially at rest (or moving much slower than m_1), then after the collision the target particle moves at twice the speed of m_1. The motion of m_1 is nearly unaffected by the collision with the much lighter target.

In alpha-particle scattering (Fig. 6-2), the incident alpha particle (whose mass is about 7000 times the electron mass) is essentially unaffected by collisions with the electrons of the target atoms (as indicated by the many straight-line paths in Fig. 6-2). The alpha particle is deflected only in the rare encounters with the massive nucleus of a target atom.

Equations 6-26 to 6-30 hold only for elastic collisions. For partially inelastic or explosive collisions, it is not possible to obtain a set of general equations like 6-24 or 6-25 for the final velocities unless we have more information about the momentum of each particle that is added or lost in the center-of-mass frame. In Chapter 11 we will see how considerations based on energy also allow us to analyze these different types of collisions. In partially inelastic or explosive collisions, the gain or loss of momentum (or energy) by one of the colliding bodies can be used to deduce properties related to the interaction between the bodies. This is a common technique used in nuclear physics, in which information about the properties of nuclei can be deduced by observing the momentum of the outgoing particles in nuclear collisions.

SAMPLE PROBLEM 6-9. An alpha particle (a nucleus of an atom of helium, $m = 4.0$ u) is accelerated to a velocity of 1.52×10^7 m/s and collides head-on with an oxygen nucleus ($m = 16.0$ u) at rest. After the collision, the oxygen nucleus moves with a velocity of 6.08×10^6 m/s along the original direction of motion of the alpha particle. (*a*) What is the velocity of the alpha particle after the collision? (*b*) Which type of collision listed in Fig. 6-17 best describes this process?

Solution (*a*) Conservation of momentum allows us to find the velocity of the alpha particle. Equation 6-19 expresses conservation of momentum for general one-dimensional collisions. We take the positive x direction to be that of the initial velocity of the alpha particle. Letting particle 1 be the alpha particle and particle 2 be the oxygen, we can write Eq. 6-19 with $v_{2ix} = 0$ as

$$v_{\alpha fx} = \frac{m_\alpha v_{\alpha ix} - m_O v_{Ofx}}{m_\alpha}$$

$$= \frac{(4.0\text{ u})(1.52 \times 10^7\text{ m/s}) - (16.0\text{ u})(6.08 \times 10^6\text{ m/s})}{4.0\text{ u}}$$

$$= -9.12 \times 10^6\text{ m/s}.$$

The alpha particle rebounds in the negative x direction.

Note that the mass units cancel in this equation, so we are free to use any convenient unit to express the masses of the particles. (*b*) The relative velocity between the lab and cm frames is given by Eq. 6-23:

$$v_x = \frac{m_\alpha v_{\alpha ix} + m_O v_{Oix}}{m_\alpha + m_O} = \frac{(4.0\text{ u})(1.52 \times 10^7\text{ m/s}) + 0}{4.0\text{ u} + 16.0\text{ u}}$$

$$= +0.304 \times 10^7\text{ m/s}.$$

The initial momentum of the alpha particle in the cm frame is then $p'_{\alpha ix} = m_\alpha v'_{\alpha ix} = m_\alpha(v_{\alpha ix} - v_x) = (4.0\text{ u})(1.52 \times 10^7\text{ m/s} - 0.304 \times 10^7\text{ m/s}) = +4.86 \times 10^7\text{ u} \cdot \text{m/s}$. The final momentum of the alpha particle is $p'_{\alpha fx} = m_\alpha v'_{\alpha fx} = m_\alpha(v_{\alpha fx} - v_x) = (4.0\text{ u})(-9.12 \times 10^6\text{ m/s} - 0.304 \times 10^7\text{ m/s}) = -4.86 \times 10^7\text{ u} \cdot \text{m/s}$. Thus $p'_{\alpha ix} = -p'_{\alpha fx}$, and the alpha particle only reverses the direction of its momentum with its magnitude unchanged. You can show that the momentum of the oxygen nucleus is also simply reversed in the collision. The reversal of the momentum of both particles, with their magnitudes remaining the same, is the characteristic of an *elastic* collision.

MULTIPLE CHOICE

6-1 Collisions

6-2 Linear Momentum

1. Which of the following objects has the largest momentum?
(A) A bullet fired from a rifle
(B) A football quarterback running at top speed
(C) A horse walking at about 2 miles/hour
(D) An elephant standing still

2. A 2-kg ball moving straight down strikes the floor at 8 m/s. It rebounds upward at 6 m/s. What is the magnitude of the change in momentum of the ball?
(A) 2 kg · m/s (B) 4 kg · m/s
(C) 14 kg · m/s (D) 28 kg · m/s

3. An object is moving in a circle at constant speed v. The magnitude of the rate of change of momentum of the object
(A) is zero. (B) is proportional to v.
(C) is proportional to v^2. (D) is proportional to v^3.

4. If the net force acting on a body is constant, what can we conclude about its momentum?
(A) The magnitude and/or the direction of $\vec{\mathbf{p}}$ may change.
(B) The magnitude of $\vec{\mathbf{p}}$ remains fixed, but its direction may change.
(C) The direction of $\vec{\mathbf{p}}$ remains fixed, but its magnitude may change.
(D) $\vec{\mathbf{p}}$ remains fixed in both magnitude and direction.

6-3 Impulse and Momentum

5. An object is moving in a circle at constant speed v. From time t_i to time t_f the object moves halfway around the circle. The magnitude of the impulse due to the net force on the object during this time interval
(A) is zero. (B) is proportional to v.
(C) is proportional to v^2. (D) is proportional to v^3.

6. If $\vec{\mathbf{J}}$ is the impulse of a particular force, what is $d\vec{\mathbf{J}}/dt$?
(A) The momentum (B) The change in momentum
(C) The force (D) The change in the force

7. A variable force acts on an object from $t_i = 0$ to t_f. The impulse of the force is zero. One can conclude that
(A) $\Delta\vec{\mathbf{r}} = 0$ and $\Delta\vec{\mathbf{p}} = 0$.
(B) $\Delta\vec{\mathbf{r}} = 0$ but possibly $\Delta\vec{\mathbf{p}} \neq 0$.
(C) possibly $\Delta\vec{\mathbf{r}} \neq 0$ but $\Delta\vec{\mathbf{p}} = 0$.
(D) possibly $\Delta\vec{\mathbf{r}} \neq 0$ and possibly $\Delta\vec{\mathbf{p}} \neq 0$.

8. A small car traveling along a road at high speed loses control. The driver has a choice—collide with a solid concrete wall or with an oncoming fully loaded 10-ton truck, also moving at high speed. Which choice results in the more serious collision? Assume in both cases that the small car is at rest after the collision.
(A) The collision with the truck
(B) The collision with the concrete wall
(C) The collisions would be equally serious, since the same impulse is imported to the car in either case.
(D) More information is needed to assess the collisions.

9. Riot police often use rubber bullets instead of lead bullets. Assume that neither bullet penetrates the skin, both have the same mass, time of contact, and initial speed. The difference is that the lead bullets "stick" while the rubber bullets bounce off. Which "hurts" more?
(A) The lead bullet
(B) The rubber bullet
(C) The bullets hurt the same amount.
(D) It depends on where you get hit.

6-4 Conservation of Momentum

10. Can the law of momentum conservation ever be violated?
(A) No
(B) Yes, if there are more than two particles
(C) Yes, when the forces between the particles are varying in time
(D) Yes, if the two particles stick together after a collision

11. A basketball player jumps up to "shoot" a basket. Is momentum conserved?
(A) Yes, but only if you choose the correct system
(B) Yes, but only in the horizontal direction
(C) No, because the velocity of the basketball player changes with time
(D) It is a bad question, because momentum conservation is for objects moving at constant speed, and the basketball player is accelerating.

6-5 Two-Body Collisions

12. Consider a one-dimensional collision that involves a body of mass m_1 originally moving in the positive x direction with speed v_0 colliding with a second body of mass m_2 originally at rest. The collision could be completely inelastic, with the two bodies sticking together, completely elastic, or somewhere in between. After the collision, m_1 moves with velocity v_1 while m_2 moves with velocity v_2.
(*a*) If $m_1 > m_2$, then
(A) $-v_0 < v_1 < 0$ (B) $0 < v_1 < v_0$
(C) $0 < v_1 < 2v_0$ (D) $v_0 < v_1 < 2v_0$
(*b*) and
(A) $-v_0 < v_2 < 0$. (B) $0 < v_2 < v_0$.
(C) $v_0/2 < v_2 < 2v_0$. (D) $v_0 < v_2 < 2v_0$.
(*c*) If $m_1 < m_2$ then
(A) $-v_0 < v_1 < 0$ (B) $-v_0 < v_1 < v_0/2$
(C) $0 < v_1 < v_0/2$ (D) $0 < v_1 < v_0$
(*d*) and
(A) $-v_0 < v_2 < 0$. (B) $-v_0 < v_2 < v_0/2$.
(C) $0 < v_2 < v_0/2$. (D) $0 < v_2 < v_0$.

QUESTIONS

1. Justify the following statement. "The law of conservation of linear momentum, as applied to a single particle, is equivalent to Newton's first law of motion."

2. A particle with mass $m = 0$ (a neutrino, possibly) carries momentum. How can this be in view of Eq. 6-1, in which we see that the momentum is directly proportional to the mass?

3. Although the acceleration of a baseball after it has been hit does not depend on who hit it, something about the baseball's flight must depend on the batter. What is it?

4. Explain how an airbag in an automobile may help to protect a passenger from serious injury in case of a collision.

5. It is said that, during a 30-mi/h collision, a 10-lb child can exert a 300-lb force against a parent's grip. How can such a large force come about?

6. Can the impulse of a force be zero, even if the force is not zero? Explain why or why not.

7. Figure 6-18 shows a popular carnival device, in which the contestant tries to see how high a weighted marker can be raised by hitting a target with a sledge hammer. What physi-

FIGURE 6-18. Question 7.

cal quantity does the device measure? Is it the average force, the maximum force, the work done, the impulse, the energy transferred, the momentum transferred, or something else? Discuss your answer.

8. An hourglass is being weighed on a sensitive balance, first when sand is dropping in a steady stream from the upper to the lower part, and then again after the upper part is empty. Are the two weights the same or not? Explain your answer.
9. Give a plausible explanation for the breaking of wooden boards or bricks by a karate punch. (See "Karate Strikes," by Jearl D. Walker, *American Journal of Physics,* October 1975, p. 845.)
10. Explain how conservation of momentum applies to a handball bouncing off a wall.
11. A football player, momentarily at rest on the field, catches a football as he is tackled by a running player on the other team. This is certainly a collision (inelastic!) and momentum must be conserved. In the reference frame of the football field, there is momentum before the collision but there seems to be none after the collision. Is linear momentum really conserved? If so, explain how. If not, explain why.
12. You are driving along a highway at 50 mi/h, followed by another car moving at the same speed. You slow to 40 mi/h but the other driver does not and there is a collision. What are the initial velocities of the colliding cars as seen from the reference frame of (*a*) yourself, (*b*) the other driver, and (*c*) a state trooper, who is in a patrol car parked by the roadside? (*d*) A judge asks whether you bumped into the other driver or the other driver bumped into you. As a physicist, how would you answer?
13. C. R. Daish has written that, for professional golfers, the initial speed of the ball off the clubhead is about 140 mi/h. He also says: (*a*) "If the Empire State Building could be swung at the ball at the same speed as the clubhead, the initial ball velocity would only be increased by about 2%" and (*b*) "once the golfer has started his or her downswing, camera clicking, sneezing, and so on can have no effect on the motion of the ball." Can you give qualitative arguments to support these two statements?
14. Two identical cubical blocks, moving in the same direction with a common speed v, strike a third such block initially at rest on a horizontal frictionless surface. What is the motion of the blocks after the collision? Does it matter whether or not the two initially moving blocks were in contact? Does it matter whether these two blocks were glued together? Assume that the collisions are (*a*) completely inelastic or (*b*) elastic.
15. In a two-body collision in the center-of-mass reference frame the momenta of the particles are equal and opposite to one another both before and after the collision. Is the line of relative motion necessarily the same after collision as before? Under what conditions would the magnitudes of the velocities of the bodies increase? decrease? remain the same as a result of the collision?

EXERCISES

6-1 Collisions

6-2 Linear Momentum

1. How fast must an 816-kg Volkswagen travel to have the same momentum as (*a*) a 2650-kg Cadillac going 16.0 km/h? (*b*) a 9080-kg truck also going 16.0 km/hr?
2. A 2000-kg truck traveling north at 40.0 km/h turns east and accelerates to 50.0 km/h. What is the magnitude and direction of the change of the truck's momentum?
3. A 4.88-kg object with a speed of 31.4 m/s strikes a steel plate at an angle of 42.0° and rebounds at the same speed and angle (Fig. 6-19). What is the change (magnitude and direction) of the linear momentum of the object?

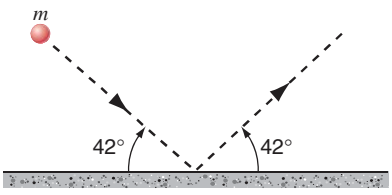


FIGURE 6-19. Exercise 3.

6-3 Impulse and Momentum

4. The bumper of a new car is being tested. The 2300-kg vehicle, moving at 15 m/s, is allowed to collide with a bridge abutment, being brought to rest in a time of 0.54 s. Find the average force that acted on the car during impact.
5. A ball of mass m and speed v strikes a wall perpendicularly and rebounds with undiminished speed. (*a*) If the time of collision is Δt, what is the average force exerted by the ball on the wall? (*b*) Evaluate this average force numerically for a rubber ball with mass 140 g moving at 7.8 m/s; the duration of the collision is 3.9 ms.
6. A golfer hits a golf ball, imparting to it an initial velocity of magnitude 52.2 m/s directed 30° above the horizontal. Assuming that the mass of the ball is 46.0 g and the club and ball are in contact for 1.20 ms, find (*a*) the impulse imparted to the ball, (*b*) the impulse imparted to the club, and (*c*) the average force exerted on the ball by the club.
7. A 150-g (weight = 5.30 oz) baseball pitched at a speed of 41.6 m/s (= 136 ft/s) is hit straight back to the pitcher at a speed of 61.5 m/s (= 202 ft/s). The bat is in contact with the ball for 4.70 ms. Find the average force exerted by the bat on the ball.

8. A force that averages 984 N is applied to a 420-g steel ball moving at 13.8 m/s by a collision lasting 27.0 ms. If the force is in a direction opposite to the initial velocity of the ball, find the final speed of the ball.

9. Figure 6-20 shows an approximate representation of force versus time during the collision of a 58-g tennis ball with a wall. The initial velocity of the ball is 32 m/s perpendicular to the wall; it rebounds with the same speed, also perpendicular to the wall. What is the value of F_{max}, the maximum contact force during the collision?

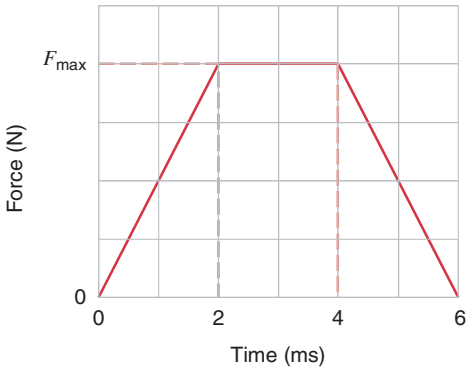


FIGURE 6-20. Exercise 9.

10. Two parts of a spacecraft are separated by detonating the explosive bolts that hold them together. The masses of the parts are 1200 kg and 1800 kg; the magnitude of the impulse delivered to each part is 300 N·s. What is the relative speed of separation of the two parts?

11. A croquet ball with a mass 0.50 kg is struck by a mallet, receiving the impulse shown in the graph (Fig. 6-21). What is the ball's velocity just after the force has become zero?

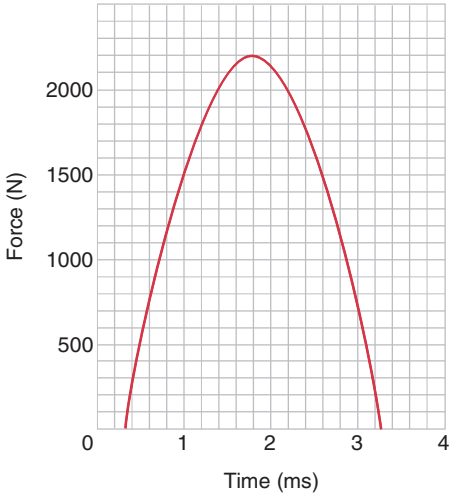


FIGURE 6-21. Exercise 11.

12. A karate expert breaks a pine board, 2.2 cm thick, with a hand chop. Strobe photography shows that the hand, whose mass may be taken as 540 g, strikes the top of the board with a speed of 9.5 m/s and comes to rest 2.8 cm below this level. (*a*) What is the time duration of the chop (assuming a constant force)? (*b*) What average force is applied?

13. A 2500-kg unmanned space probe is moving in a straight line at a constant speed of 300 m/s. A rocket engine on the space probe executes a burn in which a thrust of 3000 N acts for 65.0 s. What is the change in momentum (magnitude only) of the probe if the thrust is backward, forward, or sideways? Assume that the mass of the ejected fuel is negligible compared to the mass of the space probe.

14. A pellet gun fires ten 2.14-g pellets per second with a speed of 483 m/s. The pellets are stopped by a rigid wall. (*a*) Find the momentum of each pellet. (*b*) Calculate the average force exerted by the stream of pellets on the wall. (*c*) If each pellet is in contact with the wall for 1.25 ms, what is the average force exerted on the wall by each pellet while in contact? Why is this so different from (*b*)?

15. After launch from Earth orbit, a robot spacecraft of mass 5400 kg is coasting at constant speed halfway through its six-month flight to Mars when a NASA engineer discovers that, instead of heading for a 100-km-high orbit above the Martian surface, it is headed on a collision course directly toward the center of the planet. To correct the course, the engineer orders a short burst from the spacecraft's thrusters transverse to the direction of its motion. The thrust engines provide a constant force of 1200 N. For how long a time must the thrusters fire to achieve the correct course? Take needed data from Appendix C, and assume the distance between Earth and Mars to remain constant at its smallest possible value.

6-4 Conservation of Momentum

16. A 195-lb man standing on a surface of negligible friction kicks forward a 0.158-lb stone lying at his feet so that it acquires a speed of 12.7 ft/s. What velocity does the man acquire as a result?

17. A 75.2-kg man is riding on a 38.6-kg cart traveling at a speed of 2.33 m/s. He jumps off in such a way as to land on the ground with zero horizontal speed. Find the resulting change in the speed of the cart.

18. A railroad flatcar of weight W can roll without friction along a straight horizontal track. Initially, a man of weight w is standing on the car, which is moving to the right with speed v_0. What is the change in velocity of the car if the man runs to the left (Fig. 6-22) so that his speed relative to the car is v_{rel} just before he jumps off at the left end?

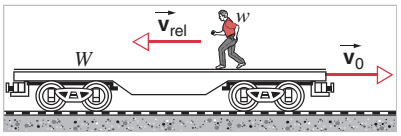


FIGURE 6-22. Exercise 18.

6-5 Two-Body Collisions

19. A space vehicle is traveling at 3860 km/h with respect to the Earth when the exhausted rocket motor is disengaged and sent backward with a speed of 125 km/h with respect to the command module. The mass of the motor is four times the mass of the module. What is the speed of the command module after the separation?

20. The blocks in Fig. 6-23 slide without friction. What is the velocity $\vec{\mathbf{v}}$ of the 1.6-kg block after the collision?

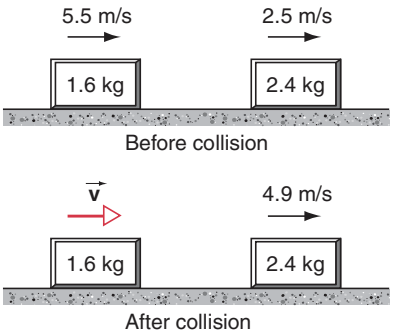


FIGURE 6-23. Exercises 20 and 21.

21. Refer to Fig. 6-23. Suppose the initial velocity of the 2.4-kg block is reversed; it is headed directly toward the 1.6-kg block. What would be the velocity $\vec{\mathbf{v}}$ of the 1.6-kg block after the collision?

22. Meteor Crater in Arizona (see Fig. 6-24) is thought to have been formed by the impact of a meteorite with the Earth some 20,000 years ago. The mass of the meteorite is estimated to be 5×10^{10} kg and its speed to have been 7.2 km/s. What speed would such a meteorite impart to the Earth in a head-on collision?

FIGURE 6-24. Exercise 22.

23. A 5.18-g bullet moving at 672 m/s strikes a 715-g wooden block at rest on a frictionless surface. The bullet emerges with its speed reduced to 428 m/s. Find the resulting speed of the block.

24. An alpha particle collides with an oxygen nucleus, initially at rest. The alpha particle is scattered at an angle of 64.0° above its initial direction of motion and oxygen nucleus recoils at an angle of 51.0° below this initial direction. The final speed of the oxygen nucleus is 1.20×10^5 m/s. What is the final speed of the alpha particle? (The mass of an alpha particle is 4.00 u and the mass of an oxygen nucleus is 16.0 u.)

25. Two objects, A and B, collide. A has mass 2.0 kg, and B has mass 3.0 kg. The velocities before the collision are $\vec{\mathbf{v}}_{iA} = (15 \text{ m/s})\hat{\mathbf{i}} + (30 \text{ m/s})\hat{\mathbf{j}}$ and $\vec{\mathbf{v}}_{iB} = (-10 \text{ m/s})\hat{\mathbf{i}} + (5.0 \text{ m/s})\hat{\mathbf{j}}$. After the collision, $\vec{\mathbf{v}}_{fA} = (-6.0 \text{ m/s})\hat{\mathbf{i}} + (30 \text{ m/s})\hat{\mathbf{j}}$. What is the final velocity of B?

26. A radioactive nucleus, initially at rest, decays by emitting an electron and a neutrino at right angles to one another. The momentum of the electron is 1.2×10^{-22} kg · m/s and that of the neutrino is 6.4×10^{-23} kg · m/s. Find the direction and magnitude of the momentum of the recoiling nucleus.

27. A barge with mass 1.50×10^5 kg is proceeding downriver at 6.20 m/s in heavy fog when it collides broadside with a barge heading directly across the river; see Fig. 6-25. The second barge has mass 2.78×10^5 kg and was moving at 4.30 m/s. Immediately after impact, the second barge finds its course deflected by 18.0° in the downriver direction and its speed increased to 5.10 m/s. The river current was practially zero at the time of the accident. What is the speed and direction of motion of the first barge immediately after the collision?

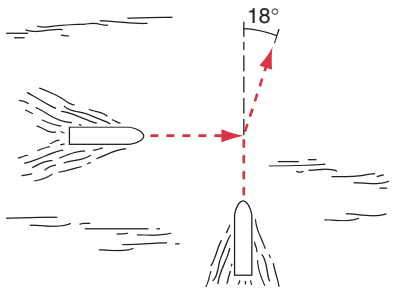


FIGURE 6-25. Exercise 27.

28. A hovering fly is approached by an enraged elephant charging at 2.1 m/s. Assuming that the collision is elastic, at what speed does the fly rebound? Note that the projectile (the elephant) is much more massive than the target (the fly).

29. Two titanium spheres approach each other head-on with the same speed and collide elastically. After the collision, one of the spheres, whose mass is 300 g, remains at rest. What is the mass of the other sphere?

30. A cart with mass 342 g moving on a frictionless linear airtrack at an initial speed of 1.24 m/s strikes a second cart of unknown mass at rest. The collision between the carts is elastic. After the collision, the first cart continues in its original direction at 0.636 m/s. (*a*) What is the mass of the second cart? (*b*) What is its speed after impact?

31. An object of 2.0-kg mass makes an elastic collision with another object at rest and continues to move in the original direction but with one-fourth of its original speed. What is the mass of the struck object?

32. A railroad freight car weighing 31.8 tons and traveling at 5.20 ft/s overtakes one weighing 24.2 tons and traveling at 2.90 ft/s in the same direction. (*a*) Find the speeds of the cars after collision if the cars couple together. (*b*) If instead, as is very unlikely, the collision is elastic, find the speeds of the cars after collision.

33. After a totally inelastic collision, two objects of the same mass and initial speed are found to move away together at half their initial speed. Find the angle between the initial velocities of the objects.

34. A proton (atomic mass 1.01 u) with a speed of 518 m/s collides elastically with another proton at rest. The original proton is scattered 64.0° from its initial direction. (*a*) What is the direction of the velocity of the target proton after the collision? (*b*) What are the speeds of the two protons after the collision?

35. In the laboratory, a particle of mass 3.16 kg moving at 15.6 m/s to the left collides head-on with a particle of mass 2.84 kg moving at 12.2 m/s to the right. Find the velocity of the center of mass of the system of two particles after the collision.

PROBLEMS

1. A stream of water impinges on a stationary "dished" turbine blade, as shown in Fig. 6-26. The speed of the water is u, both before and after it strikes the curved surface of the blade, and the mass of water striking the blade per unit time is constant at the value μ. Find the force exerted by the water on the blade.

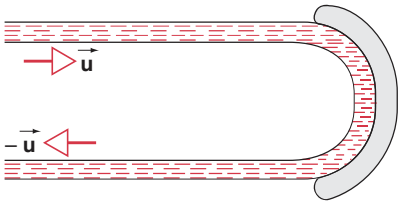


FIGURE 6-26. Problem 1.

2. A 1420-kg car moving at 5.28 m/s is initially traveling north. After completing a 90° right-hand turn in 4.60 s, the inattentive operator drives into a tree, which stops the car in 350 ms. What is the magnitude of the impulse delivered to the car (*a*) during the turn and (*b*) during the collision? What average force acts on the car (*c*) during the turn and (*d*) during the collision?

3. A 325-g ball with a speed v of 6.22 m/s strikes a wall at an angle θ of 33.0° and then rebounds with the same speed and angle (Fig. 6-27). It is in contact with the wall for 10.4 ms. (*a*) What impulse was experienced by the ball? (*b*) What was the average force exerted by the ball on the wall?

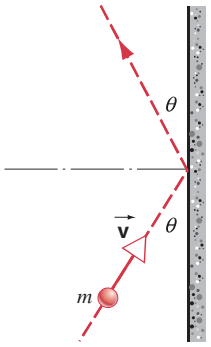


FIGURE 6-27. Problem 3.

4. It is well known that bullets and other missiles fired at Superman simply bounce off his chest as in Fig. 6-28. Suppose that a gangster sprays Superman's chest with 3.0-g bullets at the rate of 100 bullets/min, the speed of each bullet being 500 m/s. Suppose too that the bullets rebound straight back with no loss in speed. Find the average force exerted by the stream of bullets on Superman's chest.

5. During a violent thunderstorm, hail the size of marbles (diameter = 1.0 cm) falls at a speed of 25 m/s. There are estimated to be 120 hailstones per cubic meter of air. Ignore the bounce of the hail on impact. (*a*) What is the mass of each hailstone? (*b*) What force is exerted by hail on a 10 m × 20 m flat roof during the storm? Assume that, as for ice, 1.0 cm^3 of hail has a mass of 0.92 g.

FIGURE 6-28. Problem 4.

6. A very flexible uniform chain of mass M and length L is suspended from one end so that it hangs vertically, the lower end just touching the surface of a table. The upper end is suddenly released so that the chain falls onto the table and coils up in a small heap, each link coming to rest the instant it strikes the table; see Fig. 6-29. Find the force exerted by the table on the chain at any instant, in terms of the weight of chain already on the table at that moment.

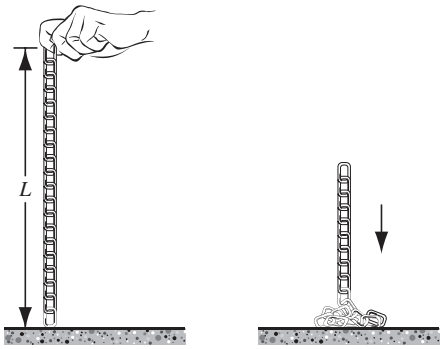


FIGURE 6-29. Problem 6.

7. A box is put on a scale that is adjusted to read zero when the box is empty. A stream of marbles is then poured into the box from a height h above its bottom at a rate of R (marbles per second). Each marble has a mass m. The collisions are completely inelastic; assume that the marbles stick to the box without bouncing when they hit. Find the scale reading of weight at time t after the marbles begin to fill the box. Determine a numerical answer when $R = 115\ s^{-1}$, $h = 9.62$ m, $m = 4.60$ g, and $t = 6.50$ s.

8. A 1930-kg railroad flatcar, which can move on the tracks with virtually no friction, is sitting motionless next to a station platform. A 108-kg football player is running along the plat-

form parallel to the tracks at 9.74 m/s. He jumps onto the back of the flatcar. (*a*) What is the speed of the flatcar after he is aboard and at rest on the flatcar? (*b*) Now he starts to walk, at 0.520 m/s relative to the flatcar, to the front of the car. What is the speed of the flatcar as he walks?

9. A 2.9-ton weight falling through a distance of 6.5 ft drives a 0.50-ton pile 1.5 in. into the ground. (*a*) Assuming that the weight–pile collision is completely inelastic, find the average force of resistance exerted by the ground. (*b*) Assuming the force of resistance by the ground remains constant at the value found in (*a*), how far into the ground would the pile be driven if the collision were elastic? (*c*) Which is more effective in this case—elastic or inelastic collisions?

10. Two 22.7-kg ice sleds are placed a short distance apart, one directly behind the other, as shown in Fig. 6-30. A 3.63-kg cat, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of 3.05 m/s relative to the sled the cat is standing on when the jump is made. Find the final speeds of the two sleds.

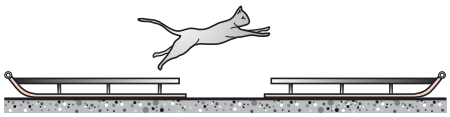


FIGURE 6-30. Problem 10.

11. Two vehicles *A* and *B* are traveling west and south, respectively, toward the same intersection where they collide and lock together. Before the collision, *A* (weight 2720 lb) is moving with a speed of 38.5 mi/h and *B* (weight 3640 lb) has a speed of 58.0 mi/h. Find the magnitude and direction of the velocity of the (interlocked) vehicles immediately after the collision.

12. Two balls *A* and *B*, having different but unknown masses, collide. *A* is initially at rest and *B* has a speed v. After collision, *B* has a speed $v/2$ and moves at right angles to its original motion. (*a*) Find the direction in which ball *A* moves after the collision. (*b*) Can you determine the speed of *A* from the information given? Explain.

13. In a game of pool, the cue ball strikes another ball initially at rest. After the collision, the cue ball moves at 3.50 m/s along a line making an angle of 65.0° with its original direction of motion. The second ball acquires a speed of 6.75 m/s. Using momentum conservation, find (*a*) the angle between the direction of motion of the second ball and the original direction of motion of the cue ball and (*b*) the original speed of the cue ball.

14. Spacecraft *Voyager 2* (mass m and speed v relative to the Sun) approaches the planet Jupiter (mass M and speed V relative to the Sun) as shown in Fig. 6-31. The spacecraft rounds the planet and departs in the opposite direction. What is its speed, relative to the Sun, after this "slingshot" encounter? Assume that $v = 12$ km/s and $V = 13$ km/s (the orbital speed of Jupiter), and that this is an elastic collision. The mass of Jupiter is very much greater than the mass of the spacecraft, $M \gg m$. (See "The Slingshot Effect: Explanation and Analogies," by Albert A. Bartlett and Charles W. Hord, *The Physics Teacher,* November 1985, p. 466.)

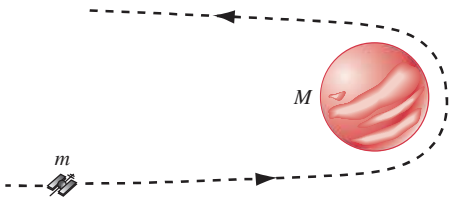


FIGURE 6-31. Problem 14.

15. The head of a golf club moving at 45.0 m/s strikes a golf ball (mass = 46.0 g) resting on a tee. The effective mass of the clubhead is 220 g. (*a*) With what speed does the ball leave the tee? (*b*) With what speed would it leave the tee if you doubled the mass of the clubhead? If you tripled it? What conclusions can you draw about the use of heavy clubs? Assume that the collisions are perfectly elastic and that the golfer can bring the heavier clubs up to the same speed at impact. See Question 13.

16. The two spheres on the right of Fig. 6-32 are slightly separated and initially at rest; the left sphere is incident with speed v_0. Assuming head-on elastic collisions, (*a*) if $M \leq m$, show that there are two collisions and find all final velocities; (*b*) if $M > m$, show that there are three collisions and find all final velocities.

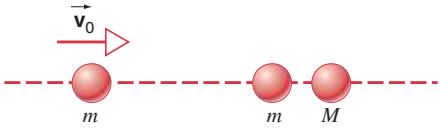


FIGURE 6-32. Problem 16.

17. A ball with an initial speed of 10.0 m/s collides elastically with two identical balls whose centers are on a line perpendicular to the initial velocity and that are initally in contact with each other (Fig. 6-33). The first ball is aimed directly at the contact point and all the balls are frictionless. Find the velocities of all three balls after the collision. (Hint: With friction absent, each impulse is directed along the line of centers of the balls, normal to the colliding surfaces.)

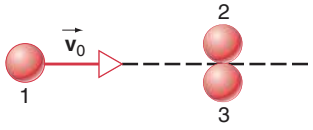


FIGURE 6-33. Problem 17.

18. Show that, in the case of an elastic collision of a particle of mass m_1 with a particle of mass m_2, initially at rest, (*a*) the maximum angle θ_m through which m_1 can be deflected by the collision is given by $\cos^2 \theta_m = 1 - (m_2/m_1)^2$, so that $0 \leq \theta_m \leq \pi/2$ when $m_1 > m_2$; (*b*) $\theta_1 + \theta_2 = \pi/2$, when $m_1 = m_2$; (*c*) θ_1 can take on all values between 0 and π when $m_1 < m_2$.

19. A 3.54-g bullet is fired horizontally at two blocks resting on a frictionless tabletop, as shown in Fig. 6-34*a*. The bullet passes through the first block, with mass 1.22 kg, and embeds itself in the second, with mass 1.78 kg. Speeds of 0.630 m/s and 1.48 m/s, respectively, are thereby imparted to the blocks, as shown in Fig. 6-34*b*. Neglecting the mass removed from the

first block by the bullet, find (*a*) the speed of the bullet immediately after emerging from the first block and (*b*) the original speed of the bullet.

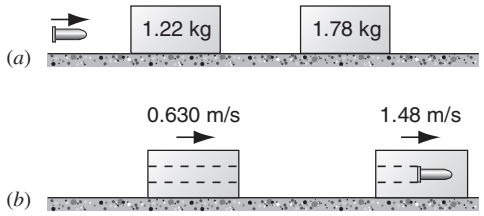


FIGURE 6-34. Problem 19.

20. A 2.0-kg block is released from rest at the top of a 22° frictionless inclined plane of height 0.65 m (Fig. 6-35). At the bottom of the plane it collides with and sticks to a block of mass 3.5 kg. The two blocks together slide a distance of 0.57 m across a horizontal plane before coming to rest. What is the coefficient of friction of the horizontal surface?

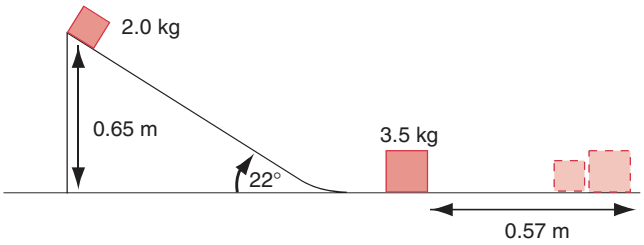


FIGURE 6-35. Problem 20.

21. Two cars *A* and *B* slide on an icy road as they attempt to stop at a traffic light. The mass of *A* is 1100 kg and the mass of *B* is 1400 kg. The coefficient of kinetic friction between the locked wheels of both cars and the road is 0.130. Car *A* succeeds in coming to rest at the light, but car *B* cannot stop and rear-ends car *A*. After the collision, *A* comes to rest 8.20 m ahead of the impact point and *B* 6.10 m ahead: see Fig. 6-36. Both drivers had their brakes locked throughout the incident. (*a*) From the distances each car moved after the collision, find the speed of each car immediately after impact. (*b*) Use conservation of momentum to find the speed at which car *B* struck car *A*. On what grounds can the use of momentum conservation be criticized here?

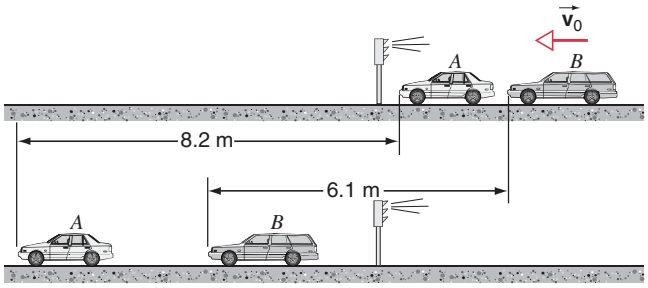


FIGURE 6-36. Problem 21.

COMPUTER PROBLEM

1. An interesting toy called the *Astro Blaster* (See Fig. 6-37) consists of four plastic balls on a stick. When the stick is dropped vertically the bottom ball bounces off the ground and then collides with the ball above. The second ball then collides with the third ball, which collides with the fourth ball. The speed of the top ball after the last collision is considerably larger than the speed at which the first ball hits the ground. Assuming all collisions are elastic, find the ratio of the masses of the four balls that will result in the largest final speed of the fourth ball, given that the lightest ball has 1/64 the mass of the heavest ball. (Note: This problem should be solved numerically, but it can also be solved analytically.)

FIGURE 6-37. Computer problem 1.